4.7 - Cauchy-Euler Equations

Definition: A **Cauchy-Euler equation** is a differential equation of the form $a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1 x \frac{dy}{dx} + a_0 y = g(x)$. Earlier in the chapter we found solutions of the form $y = e^{mx}$. Here we will find solutions of the form $y = x^m$.

Solve the given differential equation.
Example: $x^2 y'' + 3xy' - 4y = 0$

Example: $4x^2y'' + y = 0$

Example: $x^2y'' - 7xy' + 41y = 0$

A Cauchy-Euler equation can be converted to an equation with constant coefficients using the substitution $x = e^t$ and the Chain Rule.

Example: Use the substitution $x = e^t$ to transform the given Cauchy-Euler equation to a differential equation with constant coefficients. Solve the original equation by solving the new equation using techniques previously learned.

$x^2y'' - 9xy' + 25y = 0$